

Entropy spectra of black holes from resonance modes in scattering by the black holes

Yongjoon Kwon and Soonkeon Nam

Department of Physics and Research Institute of Basic Sciences,
Kyung Hee University, Seoul 130-701, Korea

E-mail: emwave@khu.ac.kr, nam@khu.ac.kr

Abstract. Since the Bekenstein's proposal that a black hole has equally spaced area spectrum, the quasinormal modes as the characteristic modes of a black hole have been used in obtaining the horizon area spectrum of the black hole. However, the area spectrum of the Kerr black hole in some previous works was inconsistent with the Bekenstein's proposal. In this paper, noting that black holes can have three types of resonance modes which are quasinormal modes (QNM), total transmission modes (TTM), and total reflection modes (TRM), we propose that all of these modes in highly damped regime should be used in quantizing the black hole. Although the QNM and the TTM of the Kerr black hole give us complicated quantization conditions from the Bohr-Sommerfeld quantization of action variable, we find a very simple result from the TRM. It gives equally spaced outer horizon area. Therefore by Bekenstein-Hawking area law we find that the Kerr black hole has universal behavior of the equally spaced entropy spectrum. With the same argument, we find that the Reissner-Nordström black hole also has equally spaced entropy spectrum.

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1. Introduction

As a quantum property of black holes, it is believed that the black hole horizon area is quantized. It was first proposed by Bekenstein [1] that the horizon area is an adiabatic invariant and should be quantized, based on Ehrenfest principle that any classical adiabatic invariant corresponds to a quantum entity with discrete spectrum. By considering the minimum change of the horizon area in the process of the assimilation of a test particle into a black hole, it was obtained that the area spectrum should be linearly quantized, i.e. $A = \gamma n \hbar$ where γ is an undetermined dimensionless constant [1]. However there are other approaches which suggest more complicated area spectrum [2] and some attempts to make a point of contact with Bekenstein's idea via proper interpretation [3].

When a perturbation on a black hole is given, the black hole undergoes damped oscillations which are called quasinormal modes. By using these quasinormal modes of a black hole, it was realized that the area spectrum of the black hole can be obtained in the semiclassical limit [4, 5, 6, 7, 8, 9, 10, 11]. By considering the real part of the asymptotic quasinormal modes of a black hole as a transition frequency in the semiclassical limit, the area spectrum of the Schwarzschild black hole was obtained as $A = (4 \ln 3) n \hbar$ [4]. However Hod's original conjecture [4] was shown to be inconsistent for other black holes [12, 13, 14]. The area spectrum of the Schwarzschild black hole was reproduced by considering an adiabatic invariant of the system with energy E and vibrational frequency $\omega(E)$, given by the real part of the asymptotic quasinormal modes, via the Bohr-Sommerfeld quantization [5]. For a rotating black hole, a modified form of adiabatic invariant was suggested as $\mathcal{I} = \int \frac{dE - \Omega dJ}{\omega(E)} = n \hbar$ via the Bohr-Sommerfeld quantization [6]. Later, it was proposed that a perturbed black hole should be described as a damped harmonic oscillator and a transition in a black hole should be considered as the transition between quantum levels $(\omega_0)_k \equiv (\sqrt{\omega_R^2 + \omega_I^2})_k$, where ω_R and ω_I are the real and imaginary parts of the asymptotic quasinormal modes [7]. Therefore the characteristic classical frequency ω_c should be identified with the transition frequency between the quantum levels $(\omega_0)_k$ in the semiclassical limit, i.e. $\omega_c = (\omega_0)_k - (\omega_0)_{k-1} \simeq (|\omega_I|)_k - (|\omega_I|)_{k-1}$ for the highly damped quasinormal modes, where $k \in N, k \gg 1$ [7]. By using this transition frequency ω_c , the area spectrum of the Schwarzschild black hole was obtained as $A = 8\pi n \hbar$ [7, 9, 10]. For the Kerr black hole, the asymptotic quasinormal modes have been first investigated numerically in Refs.[14, 15]. The transition frequency ω_c can be obtained from the asymptotic quasinormal modes which are analytically calculated as

$$\omega = \tilde{\omega}_0 - i4\pi T_0(a) (n + 1/2) , \quad (1)$$

where n is integers, $\tilde{\omega}_0$ is a function of the black hole parameters whose real part asymptotically approaches $\text{Re}(\tilde{\omega}_0) \propto m$, and $T_0(a \equiv J/M) \approx -T_H(a=0)/2$ within $\sim 3\%$ accuracy [16, 17]. The explicit expression of the imaginary part of the quasinormal modes is given in terms of the elliptic integrals [18], and it is too complicated to calculate the area spectrum by using adiabatic invariant. In the previous works [8, 9] on the area

spectrum of the Kerr black hole, with the transition frequency approximately taken as $\omega_c \approx 2\pi T_H(a=0) = 1/(4M)$, the quantization of the modified adiabatic invariant, i.e. $\mathcal{I} = \int \frac{dE - \Omega dJ}{\omega_c} = n\hbar$, was calculated. But this gives non-equally spaced area spectrum for the Kerr black hole, which is inconsistent with Bekenstein's proposal. Only for slowly rotating case with small angular momentum J compared to mass M of the Kerr black hole, it was found that the area spectrum was approximately equally spaced as $A = 8\pi n\hbar$ [9, 19]. In our recent work [20] it was reminded that an action variable is adiabatic invariant, but not every adiabatic invariant is an action variable, and that only action variable can be quantized via the Bohr-Sommerfeld quantization in the semiclassical limit. Therefore not an adiabatic invariant but an action variable of the classical system should be identified in order to apply the Bohr-Sommerfeld quantization [20]. By Bohr's correspondence principle which says that the transition frequency at large quantum number equals to the classical oscillation frequency of the corresponding classical system, a black hole with the transition frequency ω_c can be considered as the classical system of periodic motion with oscillation frequency ω_c in the semiclassical limit. Therefore the action variable of the classical periodic system with the oscillation frequency ω_c was identified and finally quantized via the Bohr-Sommerfeld quantization in the semiclassical limit as follows [20]:

$$\mathfrak{I} = \int \frac{dE}{\omega_c} = \int \frac{dM}{\omega_c} = n\hbar \quad , \quad (n \in \mathbb{Z}, |n| \gg 1) \quad , \quad (2)$$

where the transition frequency ω_c in the semiclassical limit is given by $\omega_c = (|\omega_I|)_k - (|\omega_I|)_{k-1}$ for highly damped modes, and the change of the energy E of a black hole is considered as the change of the ADM mass M (or ADT mass \mathcal{M} according to the gravity theory [21]). This formula can be also applied for a rotating black hole with a transition frequency. For example, the area and entropy spectra of the BTZ and warped AdS_3 black holes were obtained in Refs.[20, 21], where it was found that there is the universality that the entropy spectrum of a black hole is equally spaced, even though area spectrum is not equally spaced [21].

In this paper, we would like to apply the formula (2) for the Kerr black hole and to find if the entropy has the universal behavior of equally spaced spectrum. Until now, in spite of the several attempts [6, 8, 9] for the area spectrum of the Kerr black hole, the results were not consistent with Bekenstein's original proposal. Recently, by considering the scattering problem on the Kerr black hole the highly damped quasinormal modes (QNM) of the Kerr black hole, which are given by Eq.(1), were obtained from the poles of the transmission and reflection amplitudes [17]. However we notice that there are other resonance modes of the Kerr black hole, which are total transmission modes (TTM) and total reflection modes (TRM). These special modes of black holes were first considered by Chandrasekhar [22], and more investigated in some works [23, 24, 25, 26]. The TRM and TTM can be obtained from the zeros of the transmission and reflection amplitudes, respectively. In order to find out the property of a black hole, we have to do scattering experiment on the black hole. In scattering problem of a black hole, the wave equation becomes Schrödinger-like equation in quantum mechanics. Then the incident

waves from infinity are reflected and transmitted because of the effective potential which plays a role of potential barrier in quantum mechanics. So, when we consider scattering problem on a black hole, the black hole can have other resonance modes of TTM and TRM as well as QNM. The QNM only depend on the black hole parameters like mass, charge, and angular momentum which characterize a black hole. Therefore it has been considered that QNM are the characteristic modes of the black hole as a fingerprint in directly identifying the existence of a black hole [27] and carry some information about quantum structure of the black hole [7]. More specifically, it was proposed that the imaginary part of the highly damped quasinormal modes represents the energy levels as quantum structure of black hole in the semiclassical limit, so that the transition frequency between the quantum levels is corresponding to the energy of emitted quanta from black hole [7]. We notice that TTM and TRM are also the characteristic modes of a black hole, since they only depend on black hole parameters. In this sense, we propose that the highly damped TTM and TRM also have the quantum structure of a black hole as the highly damped QNM. Therefore we should promote TTM and TRM to the equivalent position of QNM, so that they play the same role as QNM in quantizing a black hole. Then we can obtain the transition frequencies corresponding to each of all of them in the semiclassical limit. By considering other resonance modes for the Kerr black hole, we will find that the area spectrum is consistent with Bekenstein's proposal [1] and the entropy spectrum has the universality of equally spaced spectrum. Moreover, with the same argument, we will also find that the Reissner-Nordström black hole has the same behavior in area and entropy spectra. Throughout this paper, the Planck units with $c = G = \hbar = 1$ are used.

2. Resonance modes of black hole in greybody factor and Hawking radiation

In this section, we will briefly review some features of resonance modes of black holes. The perturbations of black hole spacetimes are represented by the radial Schrödinger-like wave equations of the form

$$\partial_z^2 f(z) + (\omega^2 - V_z(z)) f(z) = 0, \quad (3)$$

where $z = z(r)$ is a tortoise coordinate which has the behavior of $z \sim r$ as $r \rightarrow \infty$ and $z \rightarrow -\infty$ as $r \rightarrow r_+$ with outer horizon radius r_+ . The scattering problem for the incident wave from spatial infinity gives the wavefunction as the solution of the above wave equation, which satisfy the following boundary conditions:

$$f(z)_\omega \sim \begin{cases} e^{-i\omega z} + R(\omega)e^{i\omega z} & , \quad \text{as } z \rightarrow \infty \\ T(\omega)e^{-i\omega z} & , \quad \text{as } z \rightarrow -\infty \end{cases} \quad (4)$$

where $T(\omega)$ and $R(\omega)$ are, respectively, the transmission and reflection amplitudes, and the purely ingoing wave at horizon is imposed as a boundary condition. We can also

consider the wavefunction for $-\omega$ which solves the wave equation (3) ;

$$f(z)_{-\omega} \sim \begin{cases} e^{i\omega z} + \tilde{R}(-\omega)e^{-i\omega z} & , \quad \text{as } z \rightarrow \infty \\ \tilde{T}(-\omega)e^{i\omega z} & , \quad \text{as } z \rightarrow -\infty \end{cases} \quad (5)$$

where $\tilde{T}(-\omega)$ and $\tilde{R}(-\omega)$ are some other transmission and reflection amplitudes. Then the conserved flux is given by

$$\mathcal{F} = \frac{1}{2i} (f(z)_{-\omega} \partial_z f(z)_\omega - f(z)_\omega \partial_z f(z)_{-\omega}). \quad (6)$$

By calculating the conserved flux at both limits of z , we obtain the following relation:

$$T(\omega)\tilde{T}(-\omega) + R(\omega)\tilde{R}(-\omega) = 1, \quad (7)$$

where $T(\omega)\tilde{T}(-\omega)$ represents absorption (transmission) probability which is associated with greybody factor of a black hole. By considering the scattering problem on a black hole, we can find the resonance modes of a black hole, i.e. TTM, TRM, and QNM, from the transmission and reflection probabilities. The TTM and TRM are respectively obtained from the zeros of $R(\omega)\tilde{R}(-\omega)$ and $T(\omega)\tilde{T}(-\omega)$, while the QNM corresponds to the poles of them.

In order to grasp some physical meaning of the resonance modes in some way, let us point out that there is the relation between Hawking radiation and the greybody factor with the information of the resonance modes of a black hole. It is well known that a black hole emits Hawking radiation which has the spectrum of the blackbody radiation [28]. The decay rate of a black hole at event (outer) horizon is given by [28]

$$\Gamma(\omega) = \frac{1}{e^{(\omega-m\Omega)/T_H} \mp 1} \equiv n_H(\omega). \quad (8)$$

The minus(plus) sign corresponds to bosons(fermions). This Hawking formula for emission spectrum indicates that the black hole is a thermal object. To a static observer at spatial infinity, however, the spectrum of Hawking radiation is not thermal [29]. Since the curvature of the spacetime geometry outside event horizon plays a role of potential barrier, it filters Hawking radiation in such way that some of radiation are transmitted to infinity and the rest are reflected into black hole. Therefore the decay rate at infinity is given by multiplying Hawking radiation at horizon by a factor, i.e. so called "greybody factor" which is dependent on frequency, as follows:

$$\Gamma(\omega) = \gamma(\omega)n_H(\omega) = \frac{\gamma(\omega)}{e^{(\omega-m\Omega)/T_H} \mp 1}. \quad (9)$$

It means that Hawking radiation of black hole does not 'blackbody' radiate, but 'greybody' radiate, when it is measured at infinity. When we consider the scattering problem where the incident wave originates from infinity, the greybody factor is defined as the absorption (transmission) probability of a black hole. Therefore the decay rate of Eq.(9) is given by

$$\Gamma(\omega) = \frac{\gamma(\omega)}{e^{(\omega-m\Omega)/T_H} \mp 1} = \frac{T(\omega)\tilde{T}(-\omega)}{e^{(\omega-m\Omega)/T_H} \mp 1}. \quad (10)$$

Therefore from Eq.(10) we find that the TTM is the resonance modes which give the decay rate in the form of blackbody radiation even at infinity. For the TRM, it seems that Eq.(10) vanishes since the TRM corresponds to $T(\omega)\tilde{T}(-\omega) = 0$. However, it is pointed out in Ref.[17] that the greybody factor has zeros only where n_H has poles in general, in particular for the spherical black holes in Ref.[29]. Therefore the black holes we will consider in this paper have non-vanishing emission spectrum $\Gamma(\omega)$ which is not affected by the TRM. In other words, the TRM is corresponding to the poles of the spectrum of Hawking radiation measured at outer horizon. In this sense, the TRM is associated with the outer horizon. On the other hand, the QNM is corresponding to the poles of spectrum of Hawking radiation measured at infinity.

3. Area and entropy spectra from TRM

For the quantization of the Kerr black hole, we will consider all of the resonance modes (i.e. TTM, TRM, and QNM) and apply them to the formula (2). Before that, let us consider the Schwarzschild black hole case first as a warm-up exercise. In the highly damped regime, the transmission and reflection probabilities are given by [29]

$$T(\omega)\tilde{T}(-\omega) \approx \frac{e^{\frac{\omega}{T_H^s}} - 1}{e^{\frac{\omega}{T_H^s}} + (1 + 2 \cos \pi j)} \quad \text{and} \quad R(\omega)\tilde{R}(-\omega) \approx \frac{2(1 + \cos \pi j)}{e^{\frac{\omega}{T_H^s}} + (1 + 2 \cos \pi j)}. \quad (11)$$

The scalar, electromagnetic and gravitational perturbations correspond to $j = 0$, $j = 1$ and $j = 2$, respectively. Since the effective potential in Schrödinger-like wave equation has the term of $(1 - j^2)/r^4$, it becomes singular for the electromagnetic perturbation ($j = 1$) [29]. Therefore the above results hold except for that case. For the scalar and gravitational perturbations, we can find QNM and TRM of the Schwarzschild black hole, but there is no TTM. The QNM and TRM for the highly damped modes are easily obtained as follows:

$$\omega^{QNM} = T_H^s \ln 3 - i2\pi T_H^s (k + 1/2), \quad (12)$$

$$\omega^{TRM} = -i2\pi T_H^s k, \quad (k \in \mathbb{N} \text{ and } k \gg 1) \quad (13)$$

where T_H^s is the Hawking temperature of the Schwarzschild black hole, and we take the time dependence of the wavefunction as $e^{-i\omega t}$. We find that the TRM gives the same transition frequency of $\omega_c = 2\pi T_H^s = 1/(4M)$ as one from the QNM. In most cases, QNM was enough in finding the horizon area spectrum of a black hole. But for some cases it is not sufficient. In particular for black holes with two horizons, other resonance modes may be needed to obtain two transition frequencies of a black hole, and from which the spectra of the both inner and outer horizon areas can be obtained. In our previous works [20, 21], for example, we have seen that the spectra of the both inner and outer horizon areas are obtained from the two transition frequencies which are read off from two families of the QNM.

The linearized and massless perturbation of the Kerr black hole is described by Teukolsky's equation [30]. The wave equation for the Kerr black hole can be solved

in the highly damped regime by using WKB approximation along specific contours in the complex r -plane [17]. Using the monodromy matching method along two different contours, the transmission and reflection probabilities of the Kerr black hole can be obtained [17]. Three resonance modes, which are QNM, TTM and TRM, correspond to the poles and zeros of them. The resonance modes in the highly damped regime has the form of

$$\omega^j = \tilde{\omega}^j + i4\pi T^j (n + \mu^j/4), \quad (14)$$

where n is integers, μ^j are Maslov indices and j denotes the three resonance modes. The highly damped QNM for the wavefunction with the time dependence of $e^{-i\omega t}$ is given by [17]

$$\omega^{QNM} = \tilde{\omega}_0 + i4\pi T_0 (k + 1/2), \quad (k \in N \text{ and } k \gg 1), \quad (15)$$

where $\tilde{\omega}^{QNM} \equiv \tilde{\omega}_0$ is a function of black hole parameters. Note that T_0 is negative vlaue. The $|T_0|$ is a monotonically increasing function of $a \equiv J/M$ and $T_0(a) \approx -T_H(a=0)/2$ within about 3% accuracy with $T_0(a \rightarrow 0) = -T_H(a=0)/2$. In Ref.[18], it is shown that the explicit expression of the quasinormal modes can be given in terms of the elliptic integrals. The TTM and TRM can be obtained from the exact relations between the parameters T^j and $\tilde{\omega}^j$ of the three resonance modes [17];

$$\frac{1}{2T^{TTM}} - \frac{1}{2T^{QNM}} = \frac{1}{2T^{TRM}} = \frac{1}{T_H}, \quad (16)$$

$$\frac{\tilde{\omega}^{TTM}}{2T^{TTM}} - \frac{\tilde{\omega}^{QNM}}{2T^{QNM}} = \frac{\tilde{\omega}^{TRM}}{2T^{TRM}} = \frac{m\Omega}{T_H} + i2\pi s, \quad (17)$$

where Ω is the angular velocity at horizon, T_H is the Hawking temperature of the Kerr black hole and s is the spin of the fields, i.e. gravitational ($s = -2$), electromagnetic ($s = -1$), and scalar ($s = 0$) fields. Note that T^{TTM} and T^{TRM} are positive and T^{QNM} is negative [17]. The origin of this relation can be most clearly understood when we consider the specific anti-stokes lines, along which the solutions of the wave equation are purely oscillatory, associated with each of the resonance modes.

For the quantization of the Kerr black hole, we would like to consider TRM and TTM as well as QNM in using the formula (2). However the expressions of T^{QNM} and T^{TTM} are given in very complicated forms with the elliptic integrals [18]. These give some difficulty in calculating the action variables in Eq.(2). Nevertheless, we find the area spectrum of the Kerr black hole from the other resonance modes, i.e. TRM. The interesting thing is that we simply get the highly damped TRM of the Kerr black hole from the relations (16) and (17) as follows:

$$\omega^{TRM} = m\Omega - i2\pi T_H(k - s), \quad (k \in N \text{ and } k \gg 1). \quad (18)$$

Therefore the transition frequency is given by

$$\omega_c^{TRM} = 2\pi T_H = \frac{\sqrt{M^4 - J^2}}{2M(M^2 + \sqrt{M^4 - J^2})}. \quad (19)$$

With this transition frequency, we obtain the quantization condition from the formula (2);

$$\mathfrak{I}^{TRM} = \int \frac{dM}{\omega_c^{TRM}} = M^2 + \sqrt{M^4 - J^2} = n_r \hbar, \quad (20)$$

where n_r are positive integers. Therefore, we find that the outer horizon area are quantized as follows:

$$A_{out} = 8\pi n_r \hbar. \quad (21)$$

By Bekenstein-Hawking area law [28, 31], this implies that the entropy spectrum is also equally spaced as $\Delta S = 2\pi$. We find that unlike other black hole cases with single horizon [7, 10, 11], which were studied before, where the QNM gives equally spaced outer horizon spectrum, in the case of the Kerr black hole with two horizons the TRM gives the equally spaced outer horizon spectrum. From the relation (16) for the gaps in the imaginary parts of the resonance modes which are associated with the transition frequencies of the Kerr black hole, we can find that the sum of action variables for TTM and QNM is equal to the action variable for TRM, which gives the quantization of the outer horizon area. In other words, we obtain the following relation:

$$\mathfrak{I}^{TTM} + \mathfrak{I}^{QNM} = \mathfrak{I}^{TRM} = \frac{A_{out}}{8\pi} = n_r \hbar \quad (22)$$

On account of the difficulty in calculations of action variables for TTM and TRM, it is not clear what quantities the TTM and QNM quantize. For the slowly rotating case, however, the roles of TTM and QNM become clearer. In that case, we can take $T^{QNM} \simeq -T_H(a=0)/2 = -T_H^s/2$, where T_H^s is the Hawking temperature of the Schwarzschild black hole. Using the relations (16) and (17) with this, we find the TTM and TRM, and from which the transition frequencies are given by

$$\omega_c^{QNM} = 2\pi T_H^s, \text{ and } \omega_c^\pm = \pm 4\pi T^\pm \quad (23)$$

The plus(minus) sign denotes TRM(TTM), and $T^+ \equiv T_H/2$ and $T^- \equiv T_{in}/2$ where $T_{in} = \kappa_-/(2\pi) = T_H T_H^s/(T_H - T_H^s)$ with the negative surface gravity κ_- at inner horizon [32]. Therefore we find that the action variables for three resonance modes are proportional to horizon areas as follows:

$$\mathfrak{I}^{QNM} = \frac{A_+ + A_-}{8\pi}, \text{ and } \mathfrak{I}^\pm = \pm \frac{A_\pm}{8\pi}. \quad (24)$$

where $A_+ \equiv A_{out}$ and $A_- \equiv A_{in}$. Therefore we find that the spectra of the both inner and outer horizon areas are equally spaced as $\Delta A_{out} = \Delta A_{in} = 8\pi \hbar$. By Bekenstein-Hawking area law [28, 31], the entropy spectrum is also equally spaced; $\Delta S = 2\pi$. Therefore, for slowly rotating case we find that while the quantization of the action variable for QNM is associated with the quantization of the total horizon area, the TTM and TRM lead to the quantizations of the inner horizon area and the outer horizon area, respectively. Therefore for black holes with multiple horizons, we conjecture that TRM rather than QNM is associated with the quantization of the outer horizon area. Indeed, in the previous work for the BTZ black hole with two horizons, we have seen that the

action variables for the two transition frequencies from the two families of the QNM lead to the quantization conditions of total horizon area and the difference between two horizon areas [20].

We can also consider the quantization of other black holes in this manner. The three types of resonance modes can be obtained from the zeros and poles of transmission (T) and reflection (R) amplitudes for waves traveling from spatial infinity to the black hole horizon. As an example, we consider the Reissner-Nordström black hole. The transmission and reflection probability in highly damped regime are given by [29]

$$T(\omega)\tilde{T}(-\omega) \approx \frac{e^{\frac{\omega}{T_H}} - 1}{e^{\frac{\omega}{T_H}} + 2 + 3e^{-\frac{\omega}{T_{in}}}} \quad \text{and} \quad R(\omega)\tilde{R}(-\omega) \approx 3\frac{1 + e^{-\frac{\omega}{T_{in}}}}{e^{\frac{\omega}{T_H}} + 2 + 3e^{-\frac{\omega}{T_{in}}}}, \quad (25)$$

where $T_{in} \equiv \kappa_-/(2\pi)$ with the negative surface gravity κ_- at inner horizon [13]. These are for the scalar, electromagnetic and gravitational perturbations. From these, we find that the Reissner-Nordström black hole has three resonance modes of QNM, TTM, and TRM in the highly damped regime. But, the highly damped QNM cannot be obtained algebraically from the poles of transmission T and reflection R , while the highly damped TTM and TRM are obtained as purely imaginary ones as follows:

$$\omega^{TTM} = i2\pi T_{in}(k + 1/2), \quad (26)$$

$$\omega^{TRM} = -i2\pi T_H k, \quad (k \in N \text{ and } k \gg 1). \quad (27)$$

where T_H is the Hawking temperature of the Reissner-Nordström black hole. Therefore from the above TTM and TRM the transition frequencies are obtained and by applying the formula (2) the corresponding two action variables give the following quantization conditions via Bohr-Sommerfeld quantization:

$$\mathfrak{I}^\pm = M\sqrt{M^2 - Q^2} \pm M^2 = n_\pm \hbar, \quad (28)$$

where the plus(minus) sign denotes TRM(TTM). It turns out that the TTM and TRM are associated with the quantizations of the inner horizon area and outer horizon area, respectively. It is easily found that the area and entropy spectra are given by $\Delta A_{out/in} = 8\pi\hbar$ and $\Delta S = 2\pi$. Therefore we conclude that the Schwarzschild, Kerr and Reissner-Nordström black holes have the universal behavior of equally spaced entropy spectra as $\Delta S = 2\pi$. The quantization of the inner horizon area can imply that there may be physical dynamics inside the outer horizon. For example, the Hawking radiation might happen at inner horizon [33]. To clarify the physical meaning of the inner horizon spectrum, the further investigation on the physical dynamics inside outer horizon is needed.

4. Conclusion

We calculated the area and entropy spectra of the Kerr and Reissner-Nordström black holes. For this, we noticed that there are three types of highly damped resonance modes of black holes, which are quasinormal modes (QNM), total transmission modes (TTM) and total reflection modes (TRM). We proposed that all of these modes

should be considered to carry information about quantum black hole since all they are characteristic modes of black hole, and therefore they should be used in quantizing a black hole. Based on Bohr's correspondence principle, the quantum black hole with a transition frequency at large quantum number is considered as the classical periodic system with the oscillation frequency equal to the transition frequency in the semiclassical limit. The action variable \mathfrak{J} of the classical system of periodic motion is identified and quantized via the Bohr-Sommerfeld quantization in the semiclassical limit as the formula (2). We applied this method for the Kerr and Reissner-Nordström black holes. For the Kerr black hole, even though it was hard to obtain the quantization conditions from TTM and QNM, we could find the quantization condition from TRM. From this, we obtained that the outer horizon area spectrum of the Kerr black hole is equally spaced as $\Delta A_{out} = 8\pi\hbar$, consistent with Bekenstein's proposal [1]. By Bekenstein-Hawking area law, the entropy spectrum also has equal spacing of $\Delta S = 2\pi$, which means that the Kerr black hole has the universal behavior of equally spaced entropy spectrum like other black holes in Refs.[20, 21]. For the Reissner-Nordström black hole, we found that the quantization conditions from TRM and TTM lead to the quantization of the outer and inner horizon area, respectively. Since the area spectra are equally spaced as $\Delta A_{out/in} = 8\pi\hbar$, we also found that the Reissner-Nordström black hole has the universal behavior of equally spaced entropy spectrum as $\Delta S = 2\pi$. These results agree with the quantization of the entropy spectrum obtained in different methods of Refs.[34, 35, 36]. Our results also give good examples for the claim in [21] that there is the universality that the entropy spectrum of a black hole is equally spaced. Therefore we found that the universality holds regardless of the dimension of spacetime, the presence of the angular momentum or charge, and the gravity theory. It is expected that the universal behavior of entropy spectrum would be useful for understanding and investigating a quantum nature of black holes as the first step toward quantum gravity.

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